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Animal Spirits and Investment Complementarities in a Temporary Equilibrium model with Underemployment

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Abstract

This paper proposes a sequential macroeconomic model to study investment complementarities in the presence of un(der)employment. It also incorporates some assumptions generally used in Keynesian economics like animal spirits, cash-in-advance constraints and irreversibility of investment decisions. The construction of the model puts special emphasis on the role of investment decisions in the formation of business cycles.Our results provide support to the view that wage flexibility does not promise neither to dampen expectation-driven economic fluctuations nor to shorten the adjustment period. In the fix-price equilibrium, there appears a trade-off between unemployment and output volatility. When the wage formation is less sensitive to changes in past level of unemployment, less pronounced, the output volatility and more pronounced the unemployment volatility.

"The fact that investment spending itself is part of final demand, however, leads to a certain circularity in the analysis: If firms decide to invest aggressively, then final demand will be high, justifying the rapid pace of investment; but likewise, weak investment demand can be partly self-justifying (Kiyotaki, 1988). This possibility of self-justifying optimism or pessimism inevitably creates a role for expectations and attitudes in the analysis, a point to which I will return...

...Finally, what about investor and analyst expectations and attitudes? I noted earlier that adding analyst expectations to standard investment equations helps to explain the 2000-02 investment bust, as analysts (presumably reflecting what managers told them) became suddenly more pessimistic in 2000. It is interesting that the same exercise has little effect on projections of these augmented equations for 2003-04. In particular, long-term analysts' earning projections appear to be bottoming out, and short-term forecasts are brightening. These trends, if they continue, are consistent with an upturn in investment beginning in the second half of this year."

Remarks by Governor Ben S. Bernanke

Before the Forecasters Club,NewYork April 24, 2003

1 Introduction

This paper proposes a sequential macroeconomic model to study investment complementarities in the presence of un(der)employment. It also incorporates some assumptions generally used in Keynesian economics like animal spirits, cash-in-advance constraints and irreversibility of investment decisions. The construction of the model puts special emphasis on the role of investment decisions in the formation of business cycles. The question of how investment decisions contribute to economic fluctuations has extensively debated in economics. Samuelson (1939) multiplier-accelerator model, Harrod (1936) and Hicks (1951) business cycle models were accounted as contributions of early Keynesian literature to this debate. Parallel to this line of research, Kalecki (1937) and Goodwin (1951) studied on endogenous formation of business cycles by bringing in the heterogeneity and distributional aspects. New Keynesian research agenda has followed a different strand and, in order to explain variations in business formation, elaborated the concept of strategic complementarities. The new-keynesian conception of strategic complementarities departs from the idea that possible mutual gains from coordinated actions of agents can move the economy to pareto-ranked multiple equilibria. The so-called the coordination failure literature grounds on the game-theoric assumption that returns to individual actions are increasing at the aggregate level. A non-exhausted list of stationary multiple equilibria models include input games (technological externalities) Bryant (1983, 1992) and Cooper and John (1988), transaction externalities Howitt (1985), searching (or thick market) externalities Diamond (1982). In a dynamic setting, Kiyotaki (1988) and Weil (1989) explored the role of animal spirits in the formation of business cycles in the presence of investment externalities. However there is a notional distinction between strategic complementaries and complementarity as it is used in recent literature. First one broadly[1] points up to incomplete market case where coordination is a non-market, self-fulfilling activity. The new-keynesian literature generally introduces government intervention as a substitute for coordination device[2]. Interestingly, this complements the keynesian assertion that market system does not offer a self-adjustment mechanism. On the other hand, the notion of complementarity in macroeconomics simply refers to the interdependence of individual actions, a situation of mutually reinforcing circularity. To explain more plainly, in the presence of complementarities, it is supposed that to some extend that there is a feedback (circularity) mechanism of individual actions. An expansion (contraction) of one activity leads to a certain expansion in the some other activities, and it creates, to some extend, some kind of feedback to initial expansion. In this respect, the presence (or the supposition) of complementarities does not necessarily imply a strategic (calculating others' reactions) action. It is a way of defining the structure of the economy. In this paper, we prefer to use the concept of complementarity without making any reference to strategic decisions.

As for the sequential character of the model, this paper shares the general structure of temporary equilibrium approach of Saraceno (2004). But unlike this paper, we have used for out-of-equilibrium positions, the assumptions of Cartelier (2003) for fix-price equilibrium. To see more clearly how we incorporate these hypotheses, we need to start from the structure of the model.

We assume that there is no distinction between capital good and consumption

[1]We refer to strong strategic complementarity case.

[2]see Heller(1986), Howitt(1985), Blanchard-Kiyotaki (1987) and Manning (1990)

good. Each firm uses a bundle of consumption goods produced by all other firms (except its own consumption good) as investment good[3]. The capacity choice related to capital investment is made for 1 period in advance in concordance with the time to build hypothesis. The investment good depreciates totally once used in the production process. There are just 2 components of aggregate demand, consumption demand from the consumer side and inter-firm investment demand. These hypotheses are used in the multiplier-accelerator models of Keynesian business-cycle models. Unlike new-keynesian literature on strategic complementarities, in our model, there is no aggregate externality of individual investment decision of firms. However investment complementarities are always present for this type of economy. It might be the case as in input games, like Byrant (1983) put it, a production function like Leontieff (min function), would make the aggregate production depend on the firm which choose to produce the least. We have overcome this difficulty by supposing no heterogeneity in technology and expectations. Without skipping the relevance of our model with multiplier-accelerator models, we have to note that our baseline model share similar characteristics to these model. This is due to the fact that we did not supposed any externality at the aggregate level which might change the stability of relative price adjustment.

[3]One might question again income feedback (ford) effects of the individual firm action. The literature on monopolistic competition literature neglects (except Dos Santos 1992) income feedback (ford) effects of the individual firm action. The absence of ford effect means that individual firm action is negligible with respect to overall economy. The demand addressed to each individual firm can not be changed by a strategic feedback action. This is seen as a violation of competitive market formation. Here, the individual investment demand of each firm has nothing to do with the strategic feedback mechanism. In every period, each firm must decide how much to invest in order to produce for the next period. The key point is that this kind of economy works always in the presence of investment complementarities. There is always investment demand addressed to each firm in every period. This complementarity relation gives rise to multiplier process. We will further introduce how expectations (animal spirits) enter into the model. However, it must be noted that income feedback effect is relatively negligible at individual level, but it contributes to multiplier process, if all firms share the same conjecture. Thus, in this respect, the multiplier process is related to non-heterogeneous expectations, not to strategic investment decision at individual level.

Contrary to non-heterogeneous case, not to invest (abstention) would be an option for the individual firm decision. In the case of abstention (a possible heterogeneous expectations case), the individual firm will increase its profit (cost reduction) in this period, but it will be constrained in terms of production capacity. Again it must increase its production capacity by increasing her labor input in the next period. Combined with other assumptions of the model, there may appear several other scenarios. However the focus of this paper is not on divergent behavior (strategies) of agents. Bomfim, A.N, and F.X. Diebold (1997) uses heterogeneous expectations and strategic complementarities (increasing returns at aggregate level) to analyze demand complementarities.

Instead of looking for an endogenous formation of business cycles like Kalecki (1937) and Goodwin (1951), the model incorporates conjectural change of expectations for the investment decisions of firms which might serve as an exogenous shock. The change in expectations in the sense of animal spirits[4] works through our time-to-build assumption. Our elaboration of animal spirits (expectations) is strictly related to the future aggregate demand[5]. Animal spirits have been used in the literature by Kiyotaki (1988) , Weil (1989) and Howitt and Mcafee (1992), unlike their rational expectation models, we preferred to use animal spirits with adaptive expectations[6].

As for the sequential character of the model, the model shares the basic structure of Saraceno (2004) which incorporates micro foundations to fix-price temporary equilibrium approach. However unlike his general equilibrium structure which assumes rationing in both market (goods and labor), we have adopted the approach of Cartelier (2003) which only considers disequilibrium in the labor market and allow for price dynamics with the help of Shapley-Shubik rule for out-of equilibrium positions. Nevertheless, the characterization of fix-equilibrium obliged us to further assumptions for a market mechanism which must be regarded only as an attempt. It must be noted that our attempt at establishing a sequential market structure is an exercise of constructing a decentralized market economy where monetary exchange provides trading in each market. Besides the complicated structure of the model, our main focus remained on the interaction between animal spirits (conjectural change of expectations) and the institutional aspect of wage formation in the presence of labor market disequilibrium.

Rest of the paper is organized as follows; the first part sets up the baseline model as the benchmark case, second part introduces time-to-build hypothesis and incorporates expectations which enables to establish the flex price equilibrium. The flex-price equilibrium replicates in some respect the multiplier-accelerator model of Samuelson (1939) with the exception that now animal spirits serves as a disturbance mechanism. When firms have optimistic (pessimistic) expectations for future aggre-

[4]In our model, expectations are neither treated as strategic actions, nor they exhibit higher beliefs (forecasting the forecast of others, like beauty contests). Firms simply guess future demand conditions without explicit motivation.

[5]We can also say total sales instead of aggregate demand.

[6]The reason behind this choice will be clearer when the sequence of the model is introduced.

gate demand, there appears a temporary increase (decrease) in output and profits for several periods. This effect further decays with dampened oscillations around the steady-state. Third part describes the market mechanism specific to competitive monetary fix-price equilibrium. After step by step iteration of fix-price equilibrium, we discuss the simulation results of the model by focusing on the interaction between animal spirits (conjectural change of expectations) and the institutional aspect of wage formation in the presence of labor market disequilibrium. The fourth part concludes.

2 The Benchmark Model

2.1 Consumers

There are H consumers having preferences over m consumption goods and labor, and consumption bundle covers all the goods produced by monopolistically competitive firms in the economy. For the sake of simplicity, we suppose that each firm produces only one good. Consumers have no intertemporal choice between periods and the total income of each consumer is composed of wages and dividends from firms. The maximization of the Dixit-Stiglitz (1977) utility function is given by;

$$\max U_h = l_h^{1-a} C_h^a \qquad 0 \leq a \leq 1 \quad h = 1..H \tag{1}$$
$$\text{s.t.} \sum_{i=1}^{m} p_i c_{i,h} + w l_h = I_h$$

where $C_h = m^{-\beta} \left(\sum_{i=1}^{m} c_{i,h}^{\frac{1}{1+\beta}} \right)^{1+\beta}$ with $\beta > 0$ and total expenditure is $E_h = \sum_{i=1}^{m} p_i c_{i,h}$ with the price index $P = m^{\beta} \left(\sum_{i=1}^{m} p_i^{-\frac{1}{\beta}} \right)^{-\beta}$, the income of each consumer is defined as , $I_h \equiv w + d$, where working time constraint is such that $1 \equiv l_h + l_h^s$. First order conditions yield the consumption demand for each differentiated good

$C_h = \frac{aI_h}{P} \quad c_{i,h} = \left(\frac{p_i}{P}\right)^{-\frac{1+\beta}{\beta}} \frac{aI_h}{mP}$

and the labor supply of each consumer.

$l_h = \frac{(1-a)I_h}{w}$ and $l_h^s = 1 - \frac{(1-a)I_h}{w}$

2.2 Producers

There are m goods produced by monopolistically competitive firms in the economy.

Production function

$$y_i = l_i^{\alpha} k_i^{1-\alpha} \tag{2}$$

where capital input is an composite index of consumption goods, and given as; $k_i = (m-1)^{-\beta} \left(\sum_{k \neq i} (k_i^k)^{\frac{1}{1+\beta}} \right)^{1+\beta}$ with the price index $P_{-i} = \left(\frac{1}{m-1} \sum_{k \neq i} p_k^{-\frac{1}{\beta}} \right)^{-\beta}$

The capital once used in production process depreciated totally. The composite investment index covers a variety of all the $m-1$ consumption goods produced. k_i^k denotes the investment (consumption good) demand of firm i from firm k. To keep the consistency, for the price index, P_{-i} denotes the price level for all consumption goods expect the differentiated good produced by firm i.

The cost function for firm i is such that ;

$C_i(w, l, p_k, k_i^k) = wl_i + \sum_{k \neq i} p_k k_i^k$

where total investment expenditure is $E_i = \sum_{k \neq i} p_k k_i^k$

The cost function can be expressed as

$$C_i\left(y_i, w, P_{-i}\right) = \mu^{-1} w^{\alpha} P_{-i}^{1-\alpha} y_i \tag{3}$$

where $\mu = \left((1-\alpha)^{1-\alpha} (\alpha)^{\alpha}\right)$

Firm i demands labor input

$l_i = y_i (\frac{p_{-i}}{w})^{1-\alpha} (\frac{\alpha}{1-\alpha})^{1-\alpha}$

Firm i demands capital[7] from the other $m-1$ firms

$k_i = \left(\frac{P_{-i}}{w} \frac{\alpha}{1-\alpha} \right)^{-\alpha} y_i$

Firm i receives capital demand from each $m-1$ firms

$K_{-i} = \frac{1}{m-1} p_i^{-\frac{1+\beta}{\beta}} \left(\frac{1}{w} \frac{\alpha}{1-\alpha} \right)^{-\alpha} \sum_{k \neq i} \left(\frac{1}{P_{-k}} \right)^{-\frac{1+\beta}{\beta}} (P_{-k})^{-\alpha} y_k$

Profit maximization[8] for firm i

[7] See appendix 1 for the derivation of capital demand

[8] See appendix 2 for the derivation mark-up pricing rule of monopolistically competitive firm i

$$\underset{p_i}{\text{Max}} \pi_i = p_i y_i - C(y_i, w, P_{-i}) \tag{4}$$
$$\text{s.t} \quad y_i = c_i + K_{-i}$$

F.O.C gives
$\frac{\partial \pi_i}{\partial p_i} = y_i + p_i \frac{\partial y_i}{\partial p_i} - \mu^{-1} w^{\alpha} P_{-i}^{1-\alpha} \frac{\partial y_i}{\partial p_i} = 0$
where the demand elasticity is $\frac{\partial y_i}{\partial p_i} \frac{p_i}{y_i} = -\frac{1+\beta}{\beta}$
In the symmetric equilibrium, the mark-up pricing for each firm is given by

$$\frac{w}{P} = \left(\frac{\mu}{1+\beta} \right)^{\frac{1}{\alpha}} \tag{5}$$

The profits can be expressed as a function of price and quantity
$\pi = py - \theta py = \frac{\beta}{(1+\beta)} py$
where $\theta = \frac{1}{1+\beta}$ and $1 - \theta = \frac{\beta}{1+\beta}$
the dividends are paid by m firms to H consumers ;

$$d \equiv \frac{m}{H} \frac{\beta}{(1+\beta)} py \tag{6}$$

2.3 General Equilibrium

For firm i, the aggregate demand is the sum of consumers' demands and the investment demand from $m-1$ firms
$y_i = c_i + K_{-i}$
$c_{i,h} = \left(\frac{p_i}{P}\right)^{-\frac{1+\beta}{\beta}} \frac{a(w+d)}{mP}$
$k_{-i} = \sum_{k \neq i} \frac{1}{m-1} \left(\frac{p_i}{P_{-k}} \right)^{-\frac{1+\beta}{\beta}} \left(\frac{P_{-k}}{w} \frac{\alpha}{1-\alpha} \right)^{-\alpha} y_k$
Aggregating over the H consumer and m firms using the mark-up pricing rule and dividends accrued to consumers
$c_i = \frac{aH}{m} \left(\frac{\mu}{1+\beta} \right)^{\frac{1}{\alpha}} + a \frac{\beta}{(1+\beta)} y$,and the capital demand $k = \frac{(1-\alpha)}{(1+\beta)} y$
Aggregate demand becomes
$y = c + k = \frac{aH}{m} \left(\frac{\mu}{1+\beta} \right)^{\frac{1}{\alpha}} + a \frac{\beta}{(1+\beta)} y + \frac{(1-\alpha)}{(1+\beta)} y$
Rearranging gives the aggregate demand in terms of basic parameters

$$y = \left(\frac{(1-\alpha)}{(1+\beta)}\right)^{\frac{1-\alpha}{\alpha}} \frac{aH\alpha}{m\left(\beta(1-a)+\alpha\right)} \tag{7}$$

Labor market is always at equilibrium once the goods market is at equilibrium. $ml^d = Hl_h^s$

$$l_h^s = l^d = \frac{aH\alpha}{\beta(1-a)+\alpha} \tag{8}$$

3 Time to build hypothesis and Flex-Price Equilibrium

We introduce time sequence to the benchmark model. Labor market and goods market clear sequentially in each period. For the moment, it is supposed that there appears no rationing in the labor market which equivalently means no unemployment in the Keynesian sense. As for the sequential character, the market clearing turns out to be simultaneously equilibrating both markets. The capacity choice related to capital investment is made for 1 period in advance in concordance with the time to build hypothesis. Thus, the linkage between 2 periods is only possible through the capital demand in advance and the expectations concerning the state of the next period. With the expectations about the state, we refer strictly to cost conditions and future demand.

There are just 2 components of aggregate demand, consumption demand from the consumer side and inter-firm investment demand. There is no intertemporal choice for consumers, they consume all the wages they get from the labor market. So consumer problem becomes only related to producers choice.

For the beginning of the period, it is nearly impossible for the firms to decide endogenously price, quantity and investment at once in a single-good economy. The reason behind is clear enough, the production process will take time. How can firm i start to produce without input (other firms' production). So firms purchase the single good in advance to be able to produce in the next period. It is a kind of time to build hypothesis like Hicks and others used. Only difference is the supposition that this capital (investment) totally depreciates once used in the production process.

$$y_{i,t} = {l_t}^{\alpha} k_{i,t-1}^{(1-\alpha)} \tag{9}$$

Profit function (cash-flow)

$\pi_{i,t} = p_{i,t} y_{i,t}(p_{i,t}) - w_{i,t} l_t(y_{i,t}, k_{i,t-1}) - \sum_{k \neq i} p_{k,t} k_{i,t}^k$

where $l_t(y_{i,t}, k_{i,t-1}) = {y_{i,t}}^{\frac{1}{\alpha}} k_{i,t-1}^{\frac{-(1-\alpha)}{\alpha}}$

The link between periods is the future projections (expectations) of firms

Firm i demands capital from the other $m-1$ firms

$k_{i,t-1} = \left(\frac{P_{-i,t-1}}{E_{t-1} w_t} \frac{\alpha}{1-\alpha} \right)^{-\alpha} E_{t-1} y_{i,t}(p_{i,t})$

Firm i receives capital demand from each $m-1$ firms

$k_{i,t}^k = \frac{1}{m-1} \left(\frac{p_{k,t}}{P_{-i,t}} \right)^{-\frac{1+\beta}{\beta}} \left(\frac{P_{-i,t}}{E_t w_{t+1}} \frac{\alpha}{1-\alpha} \right)^{-\alpha} E_t y_{i,t+1}(p_{i,t+1})$

The aggregate demand $y_{i,t}(p_{i,t})$ for firm i at time t becomes

$c_{i,h} = \left(\frac{p_{i,t}}{P_t} \right)^{-\frac{1+\beta}{\beta}} \frac{a(w_t+d_t)}{mP_t}$ and $k_{-i,t} = \sum_{k \neq i} \frac{1}{m-1} \left(\frac{p_{k,t}}{P_{-i,t}} \right)^{-\frac{1+\beta}{\beta}} \left(\frac{P_{-i,t}}{E_t w_{t+1}} \frac{\alpha}{1-\alpha} \right)^{-\alpha} E_t y_{i,t+1}(p_{i,t+1})$

Rewriting the profit function gives

$$\pi_{i,t} = \left(\begin{array}{c} p_{i,t} y_{i,t}(p_{i,t}) - w_{i,t} (y_{i,t}(p_{i,t}))^{\frac{1}{\alpha}} \left(\left(\frac{P_{-i,t-1}}{E_{t-1} w_t} \frac{\alpha}{1-\alpha} \right)^{-\alpha} E_{t-1} y_{i,t}(p_{i,t}) \right)^{\frac{-(1-\alpha)}{\alpha}} \\ - \sum_{k \neq i} p_{k,t} \frac{1}{m-1} \left(\frac{p_{k,t}}{P_{-i,t}} \right)^{-\frac{1+\beta}{\beta}} \left(\frac{P_{-i,t}}{E_t w_{t+1}} \frac{\alpha}{1-\alpha} \right)^{-\alpha} E_t y_{i,t+1}(p_{i,t+1}) \end{array} \right) \tag{10}$$

F.O.C yield by assuming symmetric equilibrium and with $w = 1$. Pricing[9] rule becomes

$$p_t = \left(\frac{\mu}{1+\beta} \right)^{-\frac{1}{\alpha}} \left(\frac{y_t}{E_{t-1} y_t} \right)^{\frac{1-\alpha}{\alpha}} \tag{11}$$

or $y_t = E_{t-1} y_t \left(\frac{\mu}{1+\beta} \right)^{\frac{1}{1-\alpha}} p_t^{\frac{\alpha}{1-\alpha}}$

3.1 Expectations

We suppose that firm problem consist of pricing today's output and guessing future conditions in order to decide how much to invest to be able to produce the tar-

[9]For the derivation of pricing rule, see appendix 3

geted level. We again assume that firms have no clear expectations for the targeted production level for the next period. So they expect that the production level at time $t-1$ will prevail in the next period, at time $t+1$. Thus the targeted production level is determined by a backwardly adaptive behavior. Firms have 2 kinds of 'expectations'. The first one is about how much to produce or what will be the level of production in the next period. Since it is impossible for firms to form expectations on the level of production that has not been decided yet, they will only have adaptive (backward looking) expectations for the level of production. As it is mentioned before there is no linkage between periods except the expectations of the firms about prices and the production level. Here, the investment demand will trigger the adjustment of the system.

Second kind of expectations is about the wage formation.

We introduce very simple expectations functions. For production level, they take the form of

$E_{t-1}y_{i,t} = \phi^s y_{i,t-2} \quad E_t y_{i,t+1} = \phi^e y_{i,t-1}$

and for labor market conditions ;

$E_{t-1}w_t = \chi^s w_{t-2} \quad E_t w_{t+1} = \chi^e w_{t-1}$

In the case where $\phi^s = \phi^e = 1$ and $\chi^s = \chi^e = 1$,the model returns to benchmark case where time sequence plays no role. For the case where $\phi^s = 1$ and $\chi^s = 1$ but $\phi^e \neq 1$ and $\chi^e \neq 1$, we suppose that there is a conjectural change of expectations which can be related to animal spirits[10]. We impose that the changes in expectations are strictly temporary. The reason behind this is that we want explore the effect of a one time change in the expectations. The other case is not of interest for this paper since it will result in the computation of a new equilibrium and the analysis of the characteristics of such an equilibrium. We will further discuss about how expectations play a role in the time sequence.

Rewriting again the aggregate demand

$y_{i,t} = c_{i,h} + k_{-i,t} = \frac{a(w_t+d_t)}{mP_t} + \left(\frac{p_t}{\chi^e w_{t-1}}\frac{\alpha}{1-\alpha}\right)^{-\alpha} \phi^e y_{i,t-1}$

The profit function becomes together with expectations

[10]With animal spirits, we refer strictly to the formation of expectations. The adaptive expectations of firms may change independent of expected price change. We take animal spirits as the expected level change in the targeted production. As in the formulation of expectation, $E_{t+1}[y_{i,t+1}] = \phi y_{i,t-1}$ where $\phi \neq 1$. For example where $\phi = 1.05$ means that firms expects 5 percent increase in aggregate demand. Here, the investment demand will increase by the *level* effect.

$$\pi_t = p_t y_t - y_t^{\frac{1}{\alpha}} \left(\left(\frac{\mu}{1+\beta} \right)^{-\frac{1}{\alpha}} \right)^{1-\alpha} \left(\frac{\alpha}{1-\alpha} \right)^{1-\alpha} y_{t-2}^{\frac{-(1-\alpha)}{\alpha}} - p_t^{1-\alpha} \left(\frac{\alpha}{1-\alpha} \right)^{-\alpha} \chi^{\alpha} y_{t-1}$$

3.2 Flex-Price Equilibrium

For the flex-price equilibrium, we replace the expectations together with our time-to-build hypothesis.

Rewriting again the aggregate demand
$y_{i,t} = c_{i,h} + k_{-i,t} = \frac{a(w_t+d_t)}{mP_t} + \left(\frac{p_t}{\chi^e w_{t-1}} \frac{\alpha}{1-\alpha} \right)^{-\alpha} \phi^e y_{i,t-1}$

The profit function becomes together with expectations and in symmetric equilibrium
$\pi_t = p_t y_t - y_t^{\frac{1}{\alpha}} \left(\left(\frac{\mu}{1+\beta} \right)^{-\frac{1}{\alpha}} \right)^{1-\alpha} \left(\frac{\alpha}{1-\alpha} \right)^{1-\alpha} y_{t-2}^{\frac{-(1-\alpha)}{\alpha}} - p_t^{1-\alpha} \left(\frac{\alpha}{1-\alpha} \right)^{-\alpha} \chi^{\alpha} y_{t-1}$

F.O.C yield by assuming symmetric equilibrium and with $w_{t-1} = 1$. Pricing[11] rule becomes

$$p_t = \left(\frac{\mu}{1+\beta} \right)^{-\frac{1}{\alpha}} \left(\frac{y_t}{E_{t-1} y_t} \right)^{\frac{1-\alpha}{\alpha}} \quad (12)$$

The pricing rule now depends on both past and present levels of aggregate demand. The detailed derivation of the pricing rule is given in appendix 3. When expectations are optimistic (pessimistic), inter-firm investment demand increases (decreases) which leads to an increase (decrease) in today's prices. Irreversibility of investment decisions along with adaptive expectations lead to dampened fluctuations for the flex-price equilibrium. The dampened fluctuations (oscillations) result from a combination of supply side and demand side effects. According to the model, the productive capacity related to capital input is decided 1 period in advance. By assuming adaptive expectations, the capital input becomes a backward looking variable with 2 period lag. On the other side, in the aggregate demand, inter-firm investment demand is a forward-looking variable with 1 period lag. The adjustment of prices in both markets maintains the convergence of the model to its initial position. Figure 1 shows the effect of a temporary change in expectations on the adjustment dynamics of the model.

With the internalization of inter-firm investment demand or investment complementarities, temporary change of expectations turn out to be temporarily self-

[11] For the derivation of pricing rule, see appendix 3 and for the simulation see appendix 4

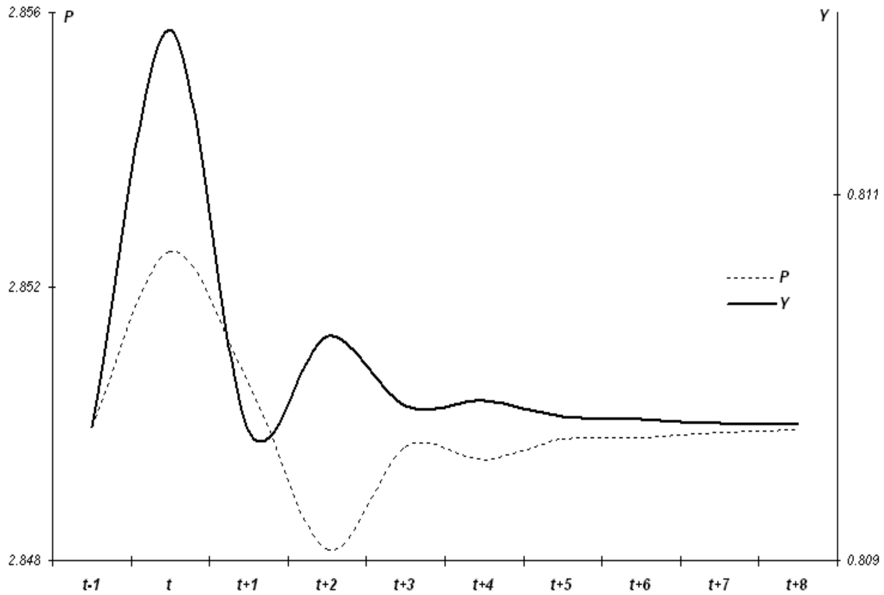

Figure 1: **Output and Price Adjustment in Flex-Price Equilibrium ($\phi = 1.05$ at time t)**

fulfilling, and after several periods, adaptive expectations and price dynamics enables to reverting to the initial position. As we have mentioned above, expectations-driven investment demand, unless there is gain from externality, produces no permanent effect on the output. Unlike the results of Kiyotaki (1988) and Weil (1989), we do not have multiple equilibria but temporary business cycle driven by animal spirits.

4 An Attempt for a Competitive Monetary Fix-Price Equilibrium

It is quiet a difficult task to establish a fix-price equilibrium, nevertheless we will attempt at describing one of the possible market mechanism that provides several features of fix-price equilibrium. The word attempt must be underlined here, since we skip some of the basic controversial issues of the literature. We will try to discuss some of them as we proceed. We suppose that there is a sequential market structure; first the labor market opens then goods market opens with the effective demand. We will further suppose that there is an inherited disequilibrium from the previous period in the labor market. The goods market clears in a way that no stocks will appear due to recursive structure of the model. Nevertheless, the sequential structure poses, besides others, 3 major difficulties. Firstly, what will be the wage adjustment mechanism when labor market is in disequilibrium position ? We do not propose any special labor market formation but along with the Keynesian literature, we suppose that the reaction of nominal wages depend on the wage-curve equation given below.

$$w_{t+1} = w_t[1 + q(U_{t-1} - U_t)] \tag{13}$$

In order to keep things simple, we suppose that wage is fixed in each period but changes according to disequilibrium over the periods. The formulation of wage-labor condition above gives the reaction of wages to changes in the unemployment rates. q denotes the speed of adjustment in the labor market which can be considered as a proxy for institutional aspect of labor market . When current (involuntary) unemployment rate is lower than the previous state, wages will increase in the next period with an adjustment speed of q[12].

Second difficulty of sequential structure is the organization of transactions[13]. How transactions will be carried over intra-period when markets open sequentially? We consider that money is the linkage between sequential markets (enables trading), and equally in a very ad-hoc fashion , we suppose that agents make their plans according to marginal analysis, but the price is supposed to be determined by means of monetary exchange. In order to set the scene for the play of monetary exchange, we suppose that there is a kind of monetary institution providing the amount of cash to carry on the desired transaction plans of agents. Obviously, this imposes some kind of constraint on the agents' plans, saying in other words, a kind of financial constraint. In more familiar terms of the monetary literature, there is a cash in advance constraint for each transaction. Money demand comes only from firms in this economy, since there is no asset market in this economy and workers can only have access to means of payments via wage income earnings.

Third difficulty is the formation of demand, it becomes crucial for the decisions

[12] We can say that this kind of wage dynamics, in some way, plays the role of Phillips curve.

[13] The organization of transactions may evoke famous failure problems. We have eliminated this kind of coordination failure problem by assuming no divergent strategy for firms. It may well be the case that some firms do not share the same conjectures as others. The model facilitates such suppositions. See Cooper and John (1987) and Bryant (1985,1990) for discussion

of firms when they first come to the labor market. Since only forward variable in the model is the inter-firm investment demand. We suppose that firms communicate with each other and each of them sends its investment schedule to others in terms of quantity signals. In the beginning of the period, each firm makes its production plan depending on consumption demand and the investment demand orders of other firms and demands money in order to carry on their transaction in both markets. We can alternatively assume that there are market specialists (retailers) calculating the demand for each good. They send their demand schedules (in terms of quantity signals) to firms in each period before the labor market opens. When labor market opens, each firm purchases labor input at the given wage in order to achieve the targeted production level. Wage are paid in nominal terms and the production capacity of each firm is restricted when labor market is closed, since capital input had already been purchased in the last period. Firms and workers come to the good market with nominal balances and transactions are settled in monetary terms. With the closing of good market, depending upon the new level of unemployment, wage adjustment takes place and next period's wage w_{t+1} is adjusted according to disequilibrium as we described above.

If there is no conjectural change in firms' expectations, the equilibrium outcome will not be different from the one in the previous period. Since there will not be a change in labor demand, the nominal wage will prevail in the next period. We will give very briefly the sequence of the market mechanism and then try to establish a competitive monetary fix-price equilibrium that enables to analyze the effects of conjectural change of expectations.

4.1 Intra-period sequence of market mechanism

The order of the sequence is given below;

1. Nominal wage is posted w_t for the period t. Firms takes the posted nominal wage as given in their calculation for future projection

2. The formation of demand with last period's aggregate demand. Firms choose their optimal investment plans and send their orders to market retailers. Now expectations of future wage are based on $E_t w_{t+1} = \chi^e w_t$ which is different from the

flex-price equilibrium since nominal wage is given at the beginning of the period. In addition to this modification, firms also take $E_t p_t = p_{t-1}$in their calculations since the price for this period is also an expected value[14]. For all market retailers, consumer demand is a forecast, which simply takes the value of the quantity of last period. Market retailers send their quantity signals which they gathered from firms and their book accounts (consumer sales of the last period).

3. Firms demand money (cash-in-advance) in terms of planned production and inter-firm payments for next period's capital demand. Firms get exactly the amount of money needed to carry their own transactions in both labor and goods market.

4. Labor market opens with the nominal wage posted at the beginning of the period, wage bills are paid in monetary terms. Workers have the money needed for their transactions in the goods market. The production takes place after the wage bill is paid.

5. Goods market open, consumers and firms come with nominal balances to the market. The production capacity is fully used in way that prevents any stocks or sub-optimal usage of factors since no factor input is storable between periods

6. Monetary price is decided at the market place in a way that matches the nominal demand and produced quantity according to the rule adopted from strategic market games, which is also called Shapley-Shubik rule.

4.2 Step by Step Iteration for a Fix-Price Equilibrium

Step by step iteration of fix-price equilibrium described above by exploiting symmetry for all firms is as follows

- Nominal wage is posted

w_t is posted for all firms so that further in the labor market only labor quantity will adjust since we supposed that capital (consumption good) had been already chosen in the previous period.

- Formation of demand

[14]Cartelier (2003) also makes a similar assumption for prices.

In order to illustrate the formation of demand described above with inter-firm investment demand, we will suppose perfect competition case to neglect the dividend problem which is more trivial[15]. So we take $\beta = 0$ and hence $d_t = 0$ and the inherited labor market disequilibrium can be described by taking in to account the fact that there is a rationing in the labor market and (some part of) the demand schedule of workers does not become effective as they had planned. We will further deal with the rationing in the labor market.

We do not change the character of optimal response of agents[16]. The aggregate demand again is composed of consumer demand and inter-firm capital demand to be used in the next period. In the symmetric equilibrium, the consumer demand for each good will be $c_t = \frac{aHw_{t-1}}{mp_{t-1}}$ which is equal to the quantity consumed in the last period.

Firms derive their investment demand similar to flex-price case, with only difference that now w_t is posted, and each firm's future cost minimization problem take account of today's wage.

$k_t = \left(\frac{p_{t-1}}{\chi^e w_t}\frac{\alpha}{1-\alpha}\right)^{-\alpha} \phi^e y_{t-1}$

In symmetric equilibrium, the aggregate demand will be the sum of consumer demand and inter-firm investment demand.

$$y_t = c_t + k_t$$

$$y_t = \frac{aHw_{t-1}}{mp_{t-1}} + \left(\frac{p_{t-1}}{\chi^e w_t}\right)^{-\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha}\phi^e y_{t-1} \tag{14}$$

Labor demand will be

$$l_t(y_t, k_{t-1}) = y_t^{\frac{1}{\alpha}} k_{t-1}^{\frac{-(1-\alpha)}{\alpha}} \tag{15}$$

where $k_{t-1} = \left(\frac{p_{t-2}}{w_{t-1}}\right)^{-\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} E_{t-1}y_t(p_t)$

rearranging and simplifying, the labor demand will be

[15] We can easily suppose that dividends are paid wit the closing of the good market and consumers demand will be $c_{i,h} = \left(\frac{p_{i,t}}{P_t}\right)^{-\frac{1+\beta}{\beta}} \frac{a(w_t+d_{t-1})}{mP_t}$.

[16] Even it would turn out to be non-optimal in the next period.

$$l_t(y_t, k_{t-1}) = \left(\frac{aHw_{t-1}}{mp_{t-1}} + \left(\frac{p_{t-1}}{\chi^e w_{t-1}} \right)^{-\alpha} \left(\frac{\alpha}{1-\alpha} \right)^{-\alpha} \phi^e y_{t-1} \right)^{\frac{1}{\alpha}} \left(\left(\frac{p_{t-2}}{w_{t-2}} \right)^{-\alpha} \left(\frac{\alpha}{1-\alpha} \right)^{-\alpha} y_{t-2} \right)^{\frac{-(1-\alpha)}{\alpha}} \tag{16}$$

- Money demand (cash-in-advance constraint)

Once the formation of demand has shaped by factor demands, for each firm, the amount of money needed to carry their transactions in the labor market and later in the goods market is also determined. The wage bill, with the posted w_t nominal wage, will be

$$w_t l_t(y_t, k_{t-1}) = w_t \left[\frac{aHw_{t-1}}{mp_{t-1}} + \left(\frac{p_{t-1}}{\chi^e w_t} \right)^{-\alpha} \left(\frac{\alpha}{1-\alpha} \right)^{-\alpha} \phi^e y_{t-1} \right]^{\frac{1}{\alpha}} \left[\left(\frac{p_{t-2}}{w_{t-1}} \right)^{-\alpha} \left(\frac{\alpha}{1-\alpha} \right)^{-\alpha} y_{t-2} \right]^{\frac{-(1-\alpha)}{\alpha}} \tag{17}$$

For each firm, expectation of future price is assumed as $E_t p_t = p_{t-1}$. The inter-firm investment demand will be

$$p_{t-1} k_t = p_{t-1} \left(\frac{p_{t-1}}{\chi^e w_t} \frac{\alpha}{1-\alpha} \right)^{-\alpha} \phi^e y_{t-1} \tag{18}$$

The money demand will be

$$M_t^d = w_t l_t(y_t, k_{t-1}) + p_{t-1} k_t \tag{19}$$

We can discuss 2 possible mechanism for monetary settlements. Firstly it can be proposed that money is not a stock variable and in each period, firms demand money in the form of credit that will be repaid once the good market closed. Along with this assumption, it is possible that firms may find themselves in monetary disequilibrium at the end of the period. There may appear a mismatch between sales receipts and monetary cost of production. In our case this monetary imbalance is not possible since firms communicate via market retailers and there is no heterogeneity in expectations. This last condition eliminates the monetary imbalance[17] for each individual firm.

[17] There is no insolvency or any kind of debtness at the end of the period for individual firms.

For a second proposition, money can be taken as a stock variable. The role of money is also limited with its use as a means of payments. Circulation of monetary stock maintains trading in each market. If there is excess (shortage) money demand, the monetary institution accommodates (withdraw) the amount of money. Stock variable or flow variable, whatever the role of money, both set-ups can be criticized with the argument that the access to means of payment is asymmetric for firms and workers. Obviously, it would be more realistic to build a more advanced credit market that would also allow workers to have access to credit for intertemporel consumption smoothing over a time horizon. In that case, we have to add a 3rd market to the model and introduce a mechanism for money market adjustments. We limit our interest with *intra-period monetary exchange* in each market. In other words, money only serves as a means of payments and is only used throughout the period[18].

Monetary institution supply the amount of money needed to carry the transactions. Note that the amount of monetary stock does not change unless there is conjectural change of expectations. Money stocks are adjusted accordingly

$$M_t^s = M_{t-1}^s + \left(M_t^d - M_{t-1}^d\right) \tag{20}$$

Firms obtain means of payments without any constraint and labor market opens with desired nominal balances.

- Labor market

Nominal wage is posted and firms come to labor market with their production plan. As we already stated, in the labor market only labor quantity will adjust since we supposed that capital (consumption good) had been already chosen in the previous period. The wage bills are paid in monetary terms and production takes place.

- Goods market

[18]For a third alternative, inter-firm payments can be settled by private notes of firms in the form of trade credit. Trade credit has been extensively studied by the literature as an important option for investment financing. See for an extensive discussion Fisman,R. Love,I. ,(2003) and Petersen, M. and Rajan, R.G, (1997)

We have come to the critical point how goods market clears under sequential structure ? It is worth noting that our assumptions create more problems than they solve. There is no easy and straight-forward mechanism like neoclassical barter (exchange of goods) for sequential economy. We will adopt a very generalized version of *pricing rule à la Shapley-Shubik* which is admitted as one of the candidate solution to the famous decentralized monetary exchange problem and proposed by Cartelier (2003) for out-of-equilibrium positions. Our adaptation of *pricing rule à la Shapley-Shubik* is even simpler than it is used in strategic market games. We suppose that firms and consumers come to goods market with their nominal balances. On the supply side, each firm had produced according to their production plan in the previous sequence. In other terms, they can not revise their production plans once goods market open. The pricing rule satisfies the match between supplied quantities and demand of firms and consumers in monetary terms and can be formulated as ;

$$p_t^s = \frac{y_t^d}{y_t^c} \tag{21}$$

where y_t^d denotes the nominal balances of firms and consumers in monetary terms, and y_t^c denotes effective (the planned level of) production. Replacing the behavioral equations in terms of parameters as described above, we have

$$p_t = \frac{\begin{array}{c} w_t \left(\frac{aHw_{t-1}}{mp_{t-1}} + \left(\frac{p_{t-1}}{\chi^e w_t}\right)^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \phi^e y_{t-1} \right)^{\frac{1}{\alpha}} \left(\left(\frac{p_{t-2}}{w_{t-1}}\right)^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} y_{t-2} \right)^{\frac{-(1-\alpha)}{\alpha}} \\ + p_{t-1} \left(\frac{p_{t-1}}{\chi^e w_t} \frac{\alpha}{1-\alpha} \right)^{-\alpha} \phi^e y_{t-1} \end{array}}{\frac{aHw_{t-1}}{mp_{t-1}} + \left(\frac{p_{t-1}}{\chi^e w_t}\right)^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \phi^e y_{t-1}} \tag{22}$$

Similar to the flex-price equilibrium, one needs a starting point in the fix-price equilibrium where there is no conjectural change expectations which is defined by $\chi^e = \phi^e = 1$. As far as factor demands and consumer demand are a function of *relative prices*, we can easily drive the pricing rule for the fix-price equilibrium when there is no conjectural change of expectations. Although, two markets open sequentially, the character of the general equilibrium will be similar to flex price under time-to-build hypothesis with only difference that now the un(der)employment appears on the labor market and adjust between periods. This assertion is more modest than it seems, since we do not deal with how unemployment emerged in this kind of economy. We simply suppose that there is an ongoing unemployment rate in this economy. Our interest is on how the adjustment takes place when facing a conjectural change of expectations. In order to make our assumptions more explicit, we will proceed what happens at the beginning of the period $t+1$.

- Opening of sequence at time $t+1$; nominal wage is posted

As we already mentioned, the inherited labor market disequilibrium can be described by taking into account the fact that there is a rationing in the labor market and (some part of) the demand schedule of workers does not become effective as they had planned. We take the unemployment parameter as $\acute{a}$ (instead of a) and short side rule determines the level of employment. This rationing can also be taken as underemployment since rationing concerns only the level of employment[19]. From previous period, at some nominal wage w_{t-1}^k[20], labor demand of firms fall short for an amount of $(a - \grave{a})$. Note that each firm faces a fixed labor supply at the competitive equilibrium in the flex-price equilibrium in the absence of dividends. We assert that in the fix-pice equilibrium, when there is no conjectural change of expectations, our competitive monetary equilibrium with rationing at some nominal wage w_{t-1}^k is not different from the competitive walrasian one under time-to-build hypothesis[21]. Workers make their optimal plans but when they come to labor market only some part $(\grave{a})$ of their optimal plan becomes effective. Our market mechanism internalizes the investment externality and effective demand of consumers in a way that maintains a fix-price equilibrium not different from the walrasian one at least for the case where expectations for future do not change.

If we impose equilibrium values for the sequence[22], the *pricing rule à la Shapley-Shubik* will be together with conditions

$$y_{t-2} = y_{t-1} = \mu^{\frac{1}{\alpha}} \frac{\grave{a}H}{m\alpha} \quad \mu = \left((1-\alpha)^{1-\alpha}\alpha^{\alpha}\right) \frac{p_{t-1}}{w_{t-1}} = \left(\frac{1}{\mu}\right)^{\frac{1}{\alpha}} \text{where } w_t = w_{t-1} = 1$$

[19]It may well be the case where rationing by number of workers which requires a modification of production function expressed in terms of not in hours but per worker

[20]which is supposed to be greater than equilibrium wage w_{t-1} so that rationing takes place

[21]See appendix for the derivation of pricing rule

[22]See appendix E

$$p_t = \frac{y_t^d}{y_t^c} = \frac{\frac{\grave{a}H}{m}\left(\left(\left(\alpha + (1-\alpha)\,\phi^e\,(\chi^e)^\alpha\right)\right)^{\frac{1}{\alpha}} + \frac{(1-\alpha)}{\alpha}\phi^e\,(\chi^e)^\alpha\right)}{\mu^{\frac{1}{\alpha}}\frac{\grave{a}H}{\alpha m}\left(\alpha + (1-\alpha)\,\phi^e\,(\chi^e)^\alpha\right)} \tag{23}$$

Note that when $\phi^e = \chi^e = 1$, the *pricing rule à la Shapley-Shubik* will be similar to the flex price under time-to-build hypothesis.

Returning to the wage adjustment mechanism described above, again when optimal behavior of consumers and firms are replaced to get what will be outcome of the rationing in the labor market. The wage adjustment at time t had been under our assumptions ;

$$w_t^k = w_{t-1}^k[1 + q(U_{t-2} - U_{t-1})]$$

where $l_{t-1}^d = l_{t-2}^d = \acute{a}H$ and $l_{t-1}^s = l_{t-2}^s = aH$. The un(der)employment rates remains unchanged with $U_{t-2} - U_{t-1} = \frac{l_{t-2}^s - l_{t-2}^d}{l_{t-2}^s} - \frac{l_{t-1}^s - l_{t-1}^d}{l_{t-1}^s} = \left(\frac{aH - \acute{a}H}{aH}\right) - \left(\frac{aH - \acute{a}H}{aH}\right) = 0$

The nominal wage at $t-1$ will also prevail at period t. As long as there is no conjectural expectation change $w_t^k = w_{t-1}^k$

Returning to labor demand, replacing the expectations and supposing $\beta = 0$, $w_{t-1}^k = 1$ and $p_{t-1} = \mu^{-\frac{1}{\alpha}}$, the labor demand will be $l_t^d = \frac{\grave{a}H}{m}\left(\left(\alpha + (1-\alpha)\,\phi^e\,(\chi^e)^\alpha\right)\right)^{\frac{1}{\alpha}}$

Together with the effective demand from investment externality, the wage dynamics will become at the end of period t.

$$w_{t+1}^k = w_t^k\left(1 + \frac{\acute{a}}{a}q\left(\left(\alpha + (1-\alpha)\,\phi^e\,(\chi^e)^\alpha\right)^{\frac{1}{\alpha}} - 1\right)\right) \tag{24}$$

where adjusted labor demand will be
$l_t^d - l_{t-1}^d = m\frac{\acute{a}H}{m}\left(\alpha + (1-\alpha)\,\phi^e\,(\chi^e)^\alpha\right)^{\frac{1}{\alpha}} - \acute{a}H = \acute{a}H\left(\left(\alpha + (1-\alpha)\,\phi^e\,(\chi^e)^\alpha\right)^{\frac{1}{\alpha}} - 1\right)$
and $U_{t-1} - U_t = \frac{l_{t-1}^s - l_{t-1}^d}{l_{t-1}^s} - \frac{l_t^s - l_t^d}{l_t^s} = \frac{\acute{a}\left(\left(\left(\alpha + (1-\alpha)\phi^e(\chi^e)^\alpha\right)\right)^{\frac{1}{\alpha}} - 1\right)}{a}$

Note that labor market adjustment is economy-wide. We aggregate labor demand of all firms. Since the nominal wage is determined according to the unemployment tension in the economy and the nominal wage is posted, afterwards the rest of the sequence follows the same routine with new demand formation and interfirm investment demand. We are able to simulate the whole model starting from where there is no conjectural change of expectations and change for some period the animal spirits of firms to see how ,under our assumptions, the system adjust in time.

4.3 Simulation Results and Discussion

Now we are able to discuss some insights of the model presented above. Certainly, the simplicity of the model does not allow to derive some very generalized implications. Nevertheless, it may help to clarify some aspects of investment complementarity. It will be useful to remind that in this type of economy where consumption goods also play the role of investment goods, the investment complementarity is internalized[23] and is always present. At every period firms must invest in order to produce for the next period. The assumption of conjectural change of expectations only serves to figure out the dynamics of such a set-up. Some kind of productivity shock or some kind of disturbance coming from consumer behavior may as well be scrutinized. But it would have lesser relevance. The outcome of the internalization of investment complementarity is certainly not a new concept and, it resonates Kalecki's dictum that "capitalists earn what they spend and workers spend what they earn". This is due to circular structure of investment complementarities. By focusing on the conjectural change of expectations, and with the help of time to build assumption, the model is able to show the dynamic effect of complementarity relations by dampened fluctuations. By construction of the model, the (short-term) multiplier process results from the circularity (feedback) of individual decisions. Unless there occurs a communication problem, the increased inter-firm investment

[23] Allowing communication (*not coordination*) in the formation of demand internalizes the externality coming from inter-firm investment demand. It may as well be the case that in the absence of perfect communication (or informational friction), firms come to the goods market without knowing *perfectly* the new investment demand schedule of other firms and consequently will not alter *perfectly* their production plan accordingly. This also refers to a (more pronounced) out-of-equilibrium position. Although, there are certain dissimilarities with his work, Cartelier (2003) explains plainly the case as " *The Shapley-Shubik rule guarantees that the corn market clears in the sense that all the quantity brought to the market is sold but at a non-expected price in general. Symmetrically, all the money spent by buyers is accepted by sellers but buyers do not generally get the quantity of corn they expected. Smith assumed such a rule in chapter 7 of the Wealth of Nations. Disequilibrium is perceived by individual agents, by contrast with Walrasian theory where individuals are always in equilibrium and where the auctioneer alone is aware of a possible disequilibrium in the market. Here, thanks to the market mechanism adopted, prices and allocations are effective in disequilibrium and not only virtual. An interesting consequence is that not only is disequilibrium dynamics meaningful but reactions to disequilibrium no longer emanate from an imaginary auctioneer but from individual agents themselves.* "

demand due to change in expectations leads to higher production level for a short period. In the flex price equilibrium, this effect is of temporary character and the model converges to initial position.

In the case of fix-price equilibrium, it totally depends on the labor market adjustment mechanism whether such expectations are fulfilled or not. A simulation exercise would provide more insights under accurate parameterization[24] for fix-price equilibrium. Although the model has a simple set-up, the concept of internalization of investment complementarity enables to discuss some of the issues related to Keynesian literature. First one, as we already underlined, is about the expectations of firms. In this economy aggregate demand is composed of current consumption demand and firms' investment demand to be used for the next period. The formation of aggregate demand is conditional on expectational investment demand of firms. Since there is no intertemporal linkage such as capital accumulation (stock problem), expectations of firms about future aggregate demand, future labor-wage conditions become crucial. We can not determine current wage and price without firms' expectations about future aggregate demand, wage and price. Without explicit expectation functions, this type of model will not work. This difficulty obliges firstly to suppose a temporary equilibrium set-up and secondly to write down explicit expectation functions. It can be said that under these assumptions, the model generates some kind of sunspot equilibria in the short run but depending on the labor market adjustment mechanism, this effect then leads to dampened fluctuations (oscillations) around the steady state.

There are 3 reasons behind these effects, first one comes from time-to-build assumption (or equivalently irreversibility of investment decisions). As expectations of firms about future demand changes, it is *immediately* internalized in today's aggregate demand formation. Once this level effect enters into today's production plan, it becomes incorporated to next period's production plan due to adaptive expectations even it is a conjectural change. Third reason is the presence of un(der)employment which is assumed to play the role of wage reaction to labor market conditions. Slow adjustment of wages is often hold responsible for keynesian unemployment to appear in models. Since we assumed away the emergence problem of unemployment,

[24]A possible extension would be the introduction of learning dynamics. For the case without communication, firms may adopt some kind of revision process, when they face unexpected profits resulting from expectational changes.

the proposed labor market adjustment mechanism only determine the volatility of dampened fluctuations around the steady state.

Before discussing the results of our simulation for fix-price equilibrium, we have to make few remarks on the labor market adjustment dynamics. The stability of the model is very sensitive to the adjustment dynamics. Designing a more complex adjustment mechanism clearly would help to enrich the model but in our simulation exercise, a further complication undermines the stability of the model. So we keep the adjustment dynamic as simple as possible like it is expressed above.

We have the model above, run for different values of q to analyze the effect of conjectural change of expectations on both unemployment and output. When q is taken as a proxy for institutional aspect of wage flexibility, it seems that there emerges a trade-off between output and unemployment volatility as it (wage flexibility q) varies. The simulation results show that as wage flexibility q increases, the output volatility also increases whereas unemployment rate fluctuations gets less pronounced. When conjectural change of expectations are optimistic, there occurs a sudden increases in output level and consequently a decrease in unemployment level. Along with adaptive expectations and wage dynamics, firms revise their optimal plans in the next periods and the effect of this sudden increase becomes temporary and fades away by reverting to the initial equilibrium position. Results of simulation exercise, Figure 2 and Figure 3 show the effect of optimistic expectations on respectively output and unemployment volatility (% deviation from initial position) for different values of wage flexibility q. The wage flexibility cast 2 effects on the output, it affects the volatility and the adjustment interval. As wage flexibility decreases, output gets less volatile and the absorption (reverting) process is faster.

However, for the unemployment volatility, it seems that intermediate levels of wage flexibility produces less volatility. Again as figure 3 shows, as wages become more flexible, the process of reverting to the initial position prolongs and oscillations are dampened less slowly.

As it is already mentioned above, the reason behind this dampened effect lies firstly in our assumption on the formation of demand. Secondly in this type of economy, the output growth is only possible through real variable like labor sup-

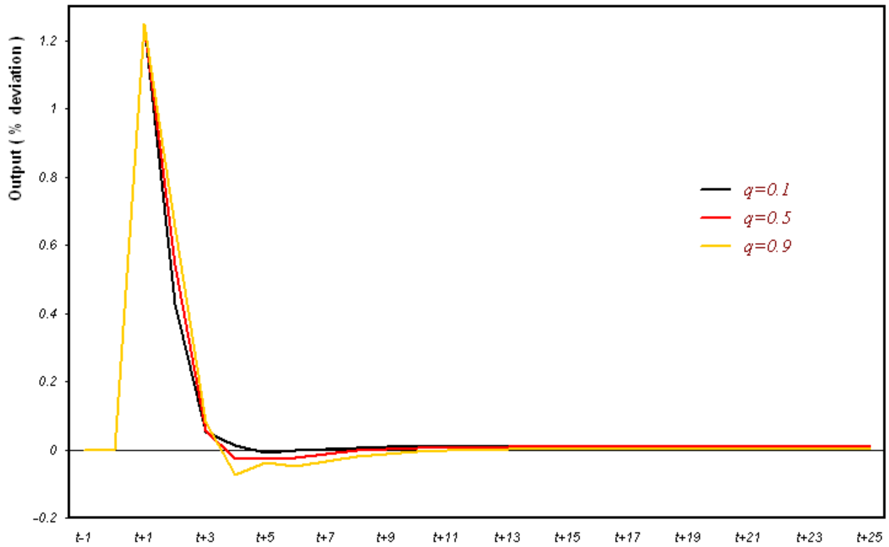

Figure 2: **Output Volatility when expectations are optimistic (**$\phi = 1.05$ **at time t+1)**

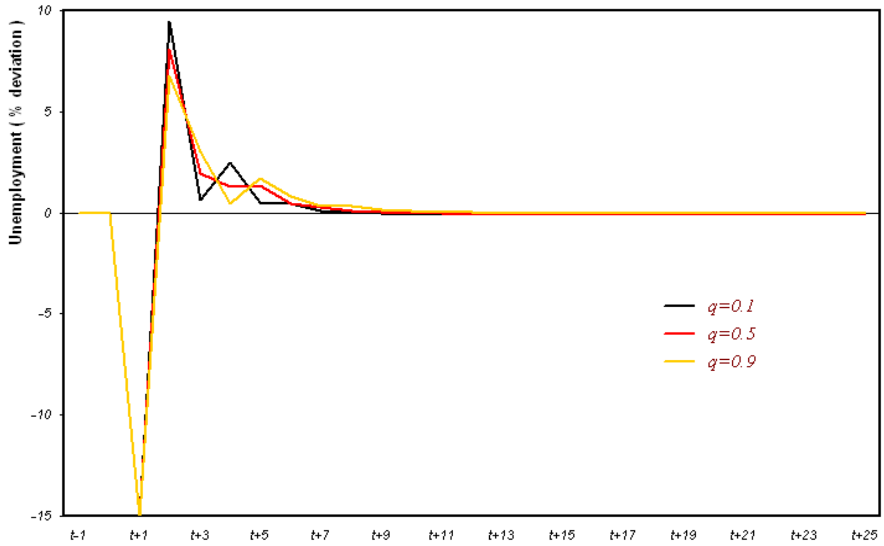

Figure 3: **Unemployment Volatility when expectations are optimistic(**$\phi = 1.05$ **at time t+1)**

ply growth and technical progress which we did not take into account in the model set-up. Besides these variables, conjectural change of expectations generates only temporary fluctuations. There is no mechanism which leads permanent changes in the cost of production. What is important here is the investment demand. In the our model, the investment demand is a function of consumer demand which is strictly linked to relative price. If relative price adjustment are not disturbed by some nominal wage dynamics [25], there is no reason to move away from initial equilibrium position even when there is underemployment in the economy. Generally, in strategic complementarity literature, it is asserted that returns to individual investment are increasing at the aggregate level, like (Kiyotaki, 1988) and Cooper and John (1987). This game-theoric formulation of investment externality might produce some distortion in price dynamics and might lead to move away from initial equilibrium position. For our case, there is no investment externality at the aggregate level, but investment complementarity is always present in every period. Once again we have to underline that the formulation of wage dynamics is crucial for the stability of the model.

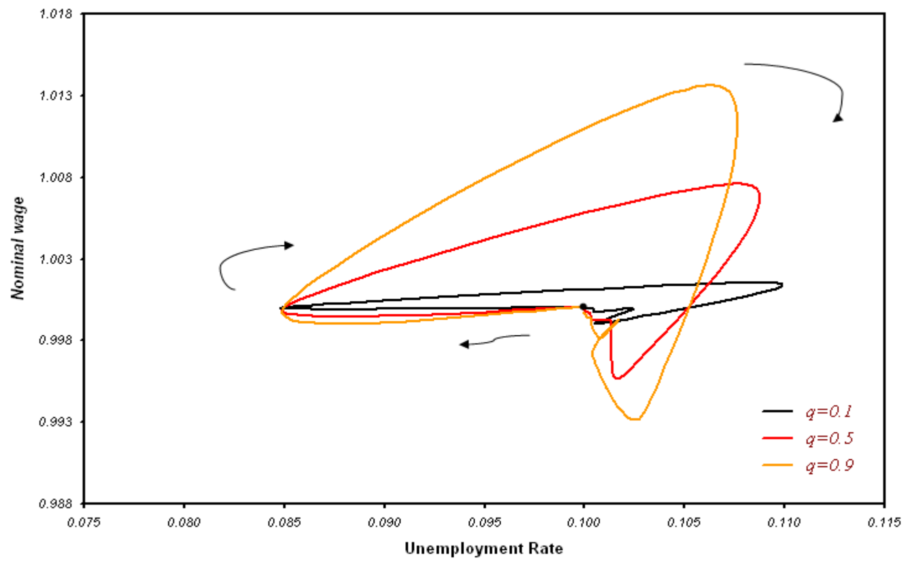

Figure 4: **Intra-period Wage-Unemployment Adjustment when expectations are optimistic($\phi = 1.05$ at time t+1)**

[25]Some nominal adjustment mechanism which is different from our formulation.

Figure 4 exhibits how wage curve reacts to change of expectations for different values of q. Again, for the case where expectations are optimistic, following the change of expectations, the unemployment-wage adjustment reverts to the initial level in the clock-wise direction. It is note worthy that throughout the period only unemployment rate adjust and wage rate reacts to changes in the labor market disequilibrium one period later. When we take the adjustment lag into consideration, the well-known downward sloping wage curve appears.

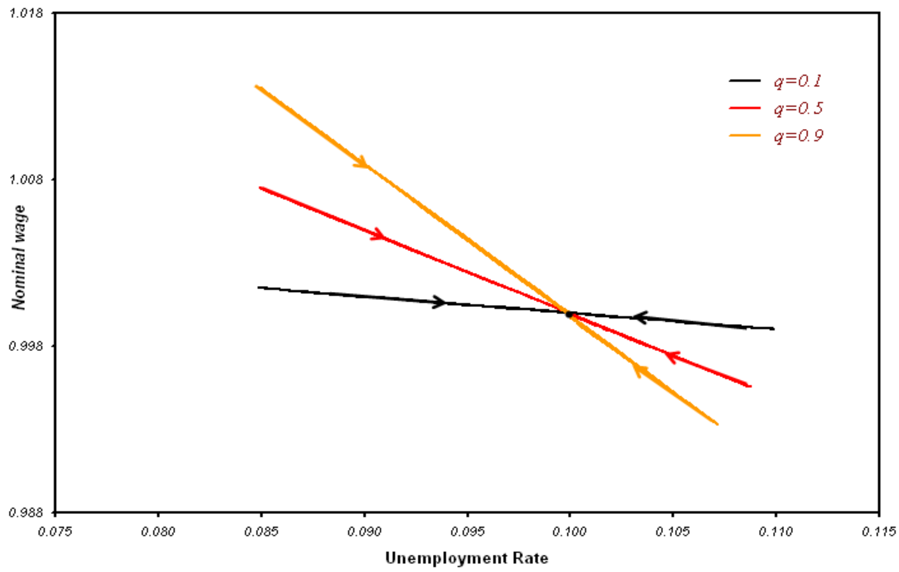

Figure 5: **Lagged Wage Curve when expectations are optimistic($\phi = 1.05$ at time t+1)**

Under some alternative formulations of the wage curve, generally, both unemployment and wage rates diverge from the initial equilibrium position in the longer run. In this respect, in the context of our simple market mechanism, instead of expectations, the destabilizing role can be attributed to institutional aspect of wage adjustment and to labor market conditions. The stability of the temporary expectations-driven fluctuations becomes strictly related to the formulation of wage adjustment in a fix-price equilibrium. It might be seen as an outside imposition to the model, but once supposed that firms have limited information of with regard to the outcome of their own actions in the presence of investment complementarities, the system might not revert to the initial position. In that case, the most probable outcome when expectations are pessimistic is the implosion of nominal (money) wages. As Keynes put it "*if money-wages were to fall without limit whenever there was a tendency for less than full employment, ... there would be no resting place below full employment until either the rate of interest was incapable of falling further,or wages were zero. In fact, we must have some factor, the value of which in terms of money is, if not fixed, at least sticky, to give us any stability of values in a monetary system*" (1936, p. 303). For Keynes, contrary to the new Keynesian literature, wage rigidity might play a stabilizing role, preventing the implosion of the system. In fact, our formulation of the wage curve, as pointed out by Fazzari and all (1998) [26], is a candidate for such implosive mechanism, since nominal wage react to changes in the level of unemployment rather than only to level of unemployment. By the construction of our model, time-to-build and single good hypotheses (consumption good as investment good) does not allow to further reductions in nominal wage. The price dynamics helps to absorb the distortion resulting from conjectural change of expectations. Although within different assumptions, Saraceno(2003) also finds that allowing for wage variability does not help reabsorb unemployment in the transitory period following an unanticipated shock. Saraceno(2003) also uses a level-dependent wage adjustment mechanism. However for the reaction of wage formation vis-a-vis the change in past levels of unemployment, our findings are similar to those of Fazzari and all (1998)[27], when wage formation is more sensitive (flexible) to labor market conditions, the output volatility decreases and the adjustment period shortens.

5 Conclusion

This paper proposes to analyze the effect of the conjectural change of expectation (animal spirits) in the presence of investment complementarities. In a flex-price approach, our results only contribute modestly to the Keynesian business cycle. Temporary changes in expectations produces temporary fluctuations and in short run, leads to partial self-fulling equilibrium with the help of multiplier process re-

[26]pp 552

[27]Their phillips-curve formulation involves a hysteresis effect for past level of unemployment. The change in level of unemployment has a dampening effect on the wage inflation. In our formulation, We prefer to use it as wage curve, or wage reaction to labor market conditions.

sulting from circularity of individual investment decisions. The system then returns to its initial position with the help of price dynamics. With reference to the new-keynesian literature on strategic complementarities, our model does not suppose any investment externality at the aggregate level, hence there is no mechanism generating multiple equilibria.

For a fix-price equilibrium, we built a market mechanism that allows sequential trading in both market. Under some restricted assumptions, we focus on the role of wage formation adjusting according to labor market conditions. Our results provide support to the view that wage flexibility does not promise neither to dampen expectation-driven economic fluctuations nor to shorten the adjustment period. However, in the fix-price equilibrium, there appears a trade-off between unemployment and output volatility. When wage formation is less sensitive to changes in past level of unemployment, less pronounced, the output volatility and more pronounced the unemployment volatility. This contradictory result can be attributed to our construction of the model which supposes inter-firm investment demand as the only forward-looking variable in accordance with time-to-build hypothesis.

For the stability of the model, we again underline the importance of wage formation as Keynes stressed as "*any stability of values in a monetary system*". In this respect, we can refer to both the literature on the endogenous business cycle models and on coordination failures , nonlinearity does not have to be searched in the formation of investment demand, the so-called destabilizing role of expectations might become effective also through labor market institutions.

6 Appendix

6.1 A The derivation of capital demand

Firm i demands capital from the other $m-1$ firms

$$k_i^k = \frac{1}{m-1}\left(\frac{p_k}{P_{-i}}\right)^{-\frac{1+\beta}{\beta}}\left(\frac{P_{-i}}{w}\frac{\alpha}{1-\alpha}\right)^{-\alpha} y_i$$

$$k_i = (m-1)^{-\beta}\left(\sum_{k\neq i}(k_i^k)^{\frac{1}{1+\beta}}\right)^{1+\beta} = (m-1)^{-\beta}\left(\sum_{k\neq i}\left(\frac{1}{m-1}\left(\frac{p_k}{P_{-i}}\right)^{-\frac{1+\beta}{\beta}}\left(\frac{P_{-i}}{w}\frac{\alpha}{1-\alpha}\right)^{-\alpha} y_i\right)^{\frac{1}{1+\beta}}\right)^{1+\beta}$$

$$k_i = (m-1)^{-\beta}\left(\left(\frac{1}{m-1}\left(\frac{1}{P_{-i}}\right)^{-\frac{1+\beta}{\beta}}\left(\frac{P_{-i}}{w}\frac{\alpha}{1-\alpha}\right)^{-\alpha} y_i\right)^{\frac{1}{1+\beta}}\sum_{k\neq i} p_k^{-\frac{1}{\beta}}\right)^{1+\beta}$$

$$k_i = \left(\frac{P_{-i}}{w}\frac{\alpha}{1-\alpha}\right)^{-\alpha} y_i \tag{25}$$

Firm i receives capital demand from each $m-1$ firms

$k_k^i = \frac{1}{m-1}\left(\frac{p_i}{P_{-k}}\right)^{-\frac{1+\beta}{\beta}} \left(\frac{P_{-k}}{w}\frac{\alpha}{1-\alpha}\right)^{-\alpha} y_k$ and $K_{-i} = \sum\limits_{k\neq i} k_k^i$

$K_{-i} = \sum\limits_{k\neq i} k_k^i = \sum\limits_{k\neq i} \frac{1}{m-1}\left(\frac{p_i}{P_{-k}}\right)^{-\frac{1+\beta}{\beta}} \left(\frac{P_{-k}}{w}\frac{\alpha}{1-\alpha}\right)^{-\alpha} y_k$

$K_{-i} = \frac{1}{m-1} p_i^{-\frac{1+\beta}{\beta}} \left(\frac{1}{w}\frac{\alpha}{1-\alpha}\right)^{-\alpha} \sum\limits_{k\neq i}\left(\frac{1}{P_{-k}}\right)^{-\frac{1+\beta}{\beta}} (P_{-k})^{-\alpha} y_k$

6.2 B Profit maximization for firm i

$$\begin{aligned} &\underset{p_i}{Max} \pi_i = p_i y_i - C\left(y_i, w, P_{-i}\right) \\ &s.t \quad y_i = c_i + K_{-i} \end{aligned} \tag{26}$$

$\frac{\partial \pi_i}{\partial p_i} = y_i + p_i \frac{\partial y_i}{\partial p_i} - \mu^{-1} w^{\alpha} P_{-i}^{1-\alpha} \frac{\partial y_i}{\partial p_i} = 0$

$-y_i = \frac{\partial y_i}{\partial p_i}\left(p_i - \mu^{-1} w^{\alpha} P_{-i}^{1-\alpha}\right)$

$-\frac{\partial p_i}{\partial y_i}\frac{y_i}{p_i} = \frac{p_i}{p_i} - \frac{\mu^{-1} w^{\alpha} P_{-i}^{1-\alpha}}{p_i}$

with $-\frac{\partial y_i}{\partial p_i}\frac{p_i}{y_i} = \epsilon$

$-\frac{1}{\varepsilon} = 1 - \frac{\mu^{-1} w^{\alpha} P_{-i}^{1-\alpha}}{p_i}$

$\frac{\mu^{-1} w^{\alpha} P_{-i}^{1-\alpha}}{p_i} = 1 + \frac{1}{\varepsilon}$

$$\frac{w}{p} = \left(\left(1 + \frac{1}{\varepsilon}\right)\mu\right)^{\frac{1}{\alpha}} \tag{27}$$

the derivation of the mark-up pricing

s.t $\quad y_i = c_i + K_{-i}$

$\quad = \left(\frac{p_i}{P}\right)^{-\frac{1+\beta}{\beta}} \frac{aHI_h}{mP} + \sum\limits_{k\neq i} \frac{1}{m-1}\left(\frac{p_i}{P_{-k}}\right)^{-\frac{1+\beta}{\beta}} \left(\frac{P_{-k}}{w}\frac{\alpha}{1-\alpha}\right)^{-\alpha} y_k$

$\frac{\partial c_i}{\partial p_i} = -\frac{1+\beta}{\beta}\frac{1}{p_i}\left(\frac{p_i}{P}\right)^{-\frac{1+\beta}{\beta}} \frac{aHI_h}{mP}$

$\frac{\partial k_{-i}}{\partial p_i} = -\frac{1+\beta}{\beta}\frac{1}{p_i}\frac{1}{m-1} p_i^{-\frac{1+\beta}{\beta}} \left(\frac{1}{w}\frac{\alpha}{1-\alpha}\right)^{-\alpha} \sum\limits_{k\neq i}\left(\frac{1}{P_{-k}}\right)^{-\frac{1+\beta}{\beta}} (P_{-k})^{-\alpha} y_k$

$\frac{\partial y_i}{\partial p_i}\frac{p_i}{y_i} = \left(\frac{\partial c_i}{\partial p_i} + \frac{\partial k_{-i}}{\partial p_i}\right)\frac{p_i}{y_i}$

$\frac{\partial y_i}{\partial p_i}\frac{p_i}{y_i} = \frac{p_i}{y_i}\left(-\frac{1+\beta}{\beta}\frac{1}{p_i}\left(\left(\frac{p_i}{P}\right)^{-\frac{1+\beta}{\beta}}\frac{aHI_h}{mP} + \frac{1}{m-1}p_i^{-\frac{1+\beta}{\beta}}\left(\frac{1}{w}\frac{\alpha}{1-\alpha}\right)^{-\alpha}\sum_{k\neq i}\left(\frac{1}{P_{-k}}\right)^{-\frac{1+\beta}{\beta}}(P_{-k})^{-\alpha}y_k\right)\right)$

$\frac{\partial y_i}{\partial p_i}\frac{p_i}{y_i} = \frac{p_i}{y_i}\frac{y_i}{p_i}\left(-\frac{1+\beta}{\beta}\right)$

Demand elasticity is ; $\frac{\partial y_i}{\partial p_i}\frac{p_i}{y_i} = -\frac{1+\beta}{\beta}$

$\theta = \frac{\varepsilon+1}{\varepsilon} = \frac{1}{1+\beta}$ and $\varepsilon = -\frac{1+\beta}{\beta}$ and $1-\theta = \frac{\beta}{1+\beta}$

For the symmetric case, the mark-up pricing is given by

$$\frac{w}{P} = \left(\frac{\mu}{1+\beta}\right)^{\frac{1}{\alpha}} \tag{28}$$

6.3 C The derivation of pricing rule *with time to build assumption*

Profit max

$\frac{\partial \pi_{i,t}}{\partial p_{i,t}} = y_{i,t}(p_{i,t}) + p_{i,t}\frac{\partial y_{i,t}(p_{i,t})}{\partial p_{i,t}} - \frac{1}{\alpha}w_{i,t}(y_{i,t}(p_{i,t}))^{\frac{1}{\alpha}-1}\frac{\partial y_{i,t}(p_{i,t})}{\partial p_{i,t}}\left(\left(\frac{P_{-i,t-1}}{E_{t-1}w_t}\frac{\alpha}{1-\alpha}\right)^{1-\alpha}(E_{t-1}y_{i,t}(p_{i,t}))^{\frac{-(1-\alpha)}{\alpha}}\right) = 0$

$y_{i,t}(p_{i,t}) + \left(\frac{\partial y_{i,t}(p_{i,t})}{\partial p_{i,t}}\right)\left[p_{i,t} - \frac{1}{\alpha}w_{i,t}(y_{i,t}(p_{i,t}))^{\frac{1}{\alpha}-1}\left(\left(\frac{P_{-i,t-1}}{E_{t-1}w_t}\frac{\alpha}{1-\alpha}\right)^{1-\alpha}(E_{t-1}y_{i,t}(p_{i,t}))^{\frac{-(1-\alpha)}{\alpha}}\right)\right] = 0$

$p_{i,t} - \frac{1}{\alpha}w_{i,t}(y_{i,t}(p_{i,t}))^{\frac{1}{\alpha}-1}\left(\left(\frac{P_{-i,t-1}}{E_{t-1}w_t}\frac{\alpha}{1-\alpha}\right)^{1-\alpha}(E_{t-1}y_{i,t}(p_{i,t}))^{\frac{-(1-\alpha)}{\alpha}}\right) = y_{i,t}(p_{i,t})\frac{\partial p_{i,t}}{\partial y_{i,t}(p_{i,t})}$

For all firms and where $w = 1$ for all past periods;

$\left(\frac{p_t}{p_t} - \frac{1}{\alpha}\frac{1}{p_t}y_t^{\frac{1}{\alpha}-1}\left(\frac{p_{t-1}}{1}\frac{\alpha}{1-\alpha}\right)^{1-\alpha}E_{t-1}y_t^{\frac{-(1-\alpha)}{\alpha}}\right) = -\frac{y_t}{p_t}\frac{1}{\frac{\partial y_t}{\partial p_t}}$

$\left(1 - \frac{1}{\alpha}\frac{1}{p_t}y_t^{\frac{1}{\alpha}-1}\left(\frac{p_{t-1}}{1}\frac{\alpha}{1-\alpha}\right)^{1-\alpha}E_{t-1}y_t^{\frac{-(1-\alpha)}{\alpha}}\right) = -\frac{1}{\varepsilon_t}$

$\frac{1}{\alpha}\frac{1}{p_t}y_t^{\frac{1}{\alpha}-1}\left(\frac{p_{t-1}}{1}\frac{\alpha}{1-\alpha}\right)^{1-\alpha}E_{t-1}y_t^{\frac{-(1-\alpha)}{\alpha}} = 1 + \frac{1}{\varepsilon_t}$ with $\varepsilon_t = -\frac{1+\beta}{\beta}$

$\frac{1}{\alpha}\frac{1}{p_t}y_t^{\frac{1}{\alpha}-1}\left(\left(\frac{p_{t-1}}{1}\frac{\alpha}{1-\alpha}\right)^{1-\alpha}E_{t-1}y_t^{\frac{-(1-\alpha)}{\alpha}}\right) = 1 + \frac{1}{-\frac{1+\beta}{\beta}}$

$y_t = p_t^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha}{1+\beta}\right)^{\frac{\alpha}{1-\alpha}}p_{t-1}^{-\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha}E_{t-1}y_t$

with from the last period $p_{t-1} = \left(\frac{\left((1-\alpha)^{1-\alpha}(\alpha)^{\alpha}\right)}{1+\beta}\right)^{-\frac{1}{\alpha}}$

$y_t = E_{t-1}y_t\left(\frac{\mu}{1+\beta}\right)^{\frac{1}{1-\alpha}}p_t^{\frac{\alpha}{1-\alpha}}$ where $\mu = \left((1-\alpha)^{1-\alpha}\alpha^{\alpha}\right)$

Pricing rule becomes

$$p_t = \left(\frac{\mu}{1+\beta}\right)^{-\frac{1}{\alpha}} \left(\frac{y_t}{E_{t-1}y_t}\right)^{\frac{1-\alpha}{\alpha}} \tag{29}$$

In order to find determine the price we have to write down the aggregate demand at time t together with expectations

Aggregate Demand with time to build assumption

The aggregate demand at time t

$c_{i,h} = \left(\frac{p_{i,t}}{P_t}\right)^{-\frac{1+\beta}{\beta}} \frac{a(w_t+d_t)}{mP_t}$ and $k_{-i,t} = \sum_{k\neq i} \frac{1}{m-1} \left(\frac{p_{k,t}}{P_{-i,t}}\right)^{-\frac{1+\beta}{\beta}} \left(\frac{P_{-i,t}}{E_t w_{t+1}} \frac{\alpha}{1-\alpha}\right)^{-\alpha} E_t y_{i,t+1}(p_{i,t+1})$

$\phi^s = 1$ and $\chi^s = 1$ $\qquad \phi^e \neq 1$ and $\chi^e \neq 1$

$E_{t-1}y_{i,t} = \phi^s y_{i,t-2}$ $\quad E_t y_{i,t+1} = \phi^e y_{i,t-1}$ $\quad E_{t-1}w_t = \chi^s w_{t-2}$ $\quad E_t w_{t+1} = \chi^e w_{t-1}$

$c_{i,h} = \frac{a(w_t+d_t)}{mP_t}$ and $k_{-i,t} = \left(\frac{p_t}{\chi^e w_{t-1}} \frac{\alpha}{1-\alpha}\right)^{-\alpha} \phi^e y_{i,t-1}$

The dividend problem

$\pi_t = p_t y_t - w_t y_t^{\frac{1}{\alpha}} (p_{t-1})^{-\alpha} \frac{\alpha}{1-\alpha}^{-\alpha} E_{t-1} y_t^{\frac{-(1-\alpha)}{\alpha}} - p_t^{1-\alpha} \left(\frac{1}{E_t w_{t+1}} \frac{\alpha}{1-\alpha}\right)^{-\alpha} E_t y_{t+1}$

$\pi_t = p_t y_t - w_t y_t^{\frac{1}{\alpha}} (p_{t-1})^{1-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} E_{t-1} y_t^{\frac{-(1-\alpha)}{\alpha}} - p_t^{1-\alpha} \left(\frac{1}{E_t w_{t+1}} \frac{\alpha}{1-\alpha}\right)^{-\alpha} E_t y_{t+1}$

$E_{t-1}y_{i,t} = \phi^s y_{i,t-2}$ $\quad E_t y_{i,t+1} = \phi^e y_{i,t-1}$ $\quad E_{t-1}w_t = \chi^s w_{t-1}$ $\quad E_t w_{t+1} = \chi^e w_t$

Replacing χ^e with χ not to complicate the notation

$\pi_t = p_t y_t - y_t^{\frac{1}{\alpha}} (p_{t-1})^{1-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} y_{t-2}^{\frac{-(1-\alpha)}{\alpha}} - p_t^{1-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \chi^\alpha \phi y_{t-1}$

$\pi_t = p_t y_t - y_t^{\frac{1}{\alpha}} \left(\left(\frac{\mu}{1+\beta}\right)^{-\frac{1}{\alpha}}\right)^{1-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} y_{t-2}^{\frac{-(1-\alpha)}{\alpha}} - p_t^{1-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \chi^\alpha y_{t-1}$

From the pricing rule $y_t = p_t^{\frac{\alpha}{1-\alpha}} \left(\frac{\alpha}{1+\beta}\right)^{\frac{\alpha}{1-\alpha}} p_{t-1}^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} E_{t-1} y_t$

$\pi_t = y_{t-2} \left(\frac{\mu}{1+\beta}\right)^{\frac{1}{1-\alpha}} p_t^{\frac{1}{1-\alpha}} - y_{t-2} \left(\frac{\mu}{1+\beta}\right)^{\frac{1}{(1-\alpha)\alpha}} p_t^{\frac{1}{1-\alpha}} \left(\frac{\mu}{1+\beta}\right)^{-\frac{1-\alpha}{\alpha}} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} - p_t^{1-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \chi^\alpha y_{t-2}$

where $y_{t-2} = \left(\frac{\mu}{(1+\beta)}\right)^{\frac{1-\alpha}{\alpha}} \frac{aH}{m(\beta(1-a)+\alpha)} (1-\alpha)^{1-\alpha} \alpha^\alpha$

Inserting dividends into the aggregate demand

$c_{i,h} = \frac{aw_t}{mP_t} + \frac{ad_t}{mP_t}$ and $k_{-i,t} = p_t^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \chi^\alpha \phi y_{i,t-1}$ $\quad d_t \equiv \frac{m}{H}\pi_t$

$y_t^d = Hc_{h,t} + k_{-i,t} = \frac{aHw_t}{mP_t} + \frac{aHd_t}{mP_t} + p_t^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \chi^\alpha \phi y_{i,t-1} = \frac{aH}{mP_t} + \frac{aH\frac{m}{H}\pi_t}{mP_t} + p_t^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \chi^\alpha \phi y_{i,t-1}$

$y_t^d = \frac{aH}{mP_t} + \frac{a\pi_t}{P_t} + p_t^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \chi^\alpha \phi y_{i,t-1}$

$y_t^d = \frac{aH}{mP_t} + \frac{a(y_{t-2})\left(\left(\frac{\mu}{1+\beta}\right)^{\frac{1}{1-\alpha}} p_t^{\frac{1}{1-\alpha}} - \left(\frac{\mu}{1+\beta}\right)^{\frac{1}{(1-\alpha)\alpha}} p_t^{\frac{1}{1-\alpha}} \left(\frac{\mu}{1+\beta}\right)^{-\frac{1-\alpha}{\alpha}} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} - p_t^{1-\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha}\chi^\alpha\phi\right)}{P_t} + p_t^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \chi^\alpha \phi y_{i,t-1}$

$y_t^d = \frac{aH}{mP_t} + a(y_{t-2}) \left(\frac{\mu}{1+\beta}\right)^{\frac{1}{1-\alpha}} p_t^{\frac{\alpha}{1-\alpha}} - a(y_{t-2}) \left(\frac{\mu}{1+\beta}\right)^{\frac{1}{(1-\alpha)\alpha}} p_t^{\frac{\alpha}{1-\alpha}} \left(\frac{\mu}{1+\beta}\right)^{-\frac{1-\alpha}{\alpha}} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} +$

$(1-a)\left(y_{t-2}\right) p_t^{-\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \chi^\alpha \phi$

Goods market equilibrium satisfies

$y_t^d = y_t^s$

$y_t^s = y_{t-2}\left(\frac{\mu}{1+\beta}\right)^{\frac{1}{1-\alpha}} p_t^{\frac{\alpha}{1-\alpha}}$

$y_{t-2}\left(\frac{\mu}{1+\beta}\right)^{\frac{1}{1-\alpha}} p_t^{\frac{\alpha}{1-\alpha}} = \frac{aH}{mP_t} + a\left(y_{t-2}\right)\left(\frac{\mu}{1+\beta}\right)^{\frac{1}{1-\alpha}} p_t^{\frac{\alpha}{1-\alpha}} - a\left(y_{t-2}\right)\left(\frac{\mu}{1+\beta}\right)^{\frac{1}{(1-\alpha)\alpha}} p_t^{\frac{\alpha}{1-\alpha}}\left(\frac{\mu}{1+\beta}\right)^{-\frac{1-\alpha}{\alpha}}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}$
$(1-a)\left(y_{t-2}\right) p_t^{-\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \chi^\alpha \phi$

$\frac{aH}{mP_t} = (1-a)\, y_{t-2}\left(\frac{\mu}{1+\beta}\right)^{\frac{1}{1-\alpha}} p_t^{\frac{\alpha}{1-\alpha}} + a\left(y_{t-2}\right)\left(\frac{\mu}{1+\beta}\right)^{\frac{1}{(1-\alpha)\alpha}} p_t^{\frac{\alpha}{1-\alpha}}\left(\frac{\mu}{1+\beta}\right)^{-\frac{1-\alpha}{\alpha}}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} -$
$(1-a)\left(y_{t-2}\right) p_t^{-\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \chi^\alpha \phi$

$\frac{aH}{mP_t}\frac{1}{y_{t-2}} = (1-a)\left(\frac{\mu}{1+\beta}\right)^{\frac{1}{1-\alpha}} p_t^{\frac{\alpha}{1-\alpha}} + a\left(\frac{\mu}{1+\beta}\right)^{\frac{1}{(1-\alpha)\alpha}} p_t^{\frac{\alpha}{1-\alpha}}\left(\frac{\mu}{1+\beta}\right)^{-\frac{1-\alpha}{\alpha}}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} - (1-a)\, p_t^{-\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \chi$

where $y_{t-2} = \left(\frac{\mu}{(1+\beta)}\right)^{\frac{1-\alpha}{\alpha}} \frac{aH}{m(\beta(1-a)+\alpha)}\mu$

$$\left(\frac{aH}{mP_t}\frac{1}{\left(\frac{\mu}{(1+\beta)}\right)^{\frac{1-\alpha}{\alpha}}\frac{aH}{m(\beta(1-a)+\alpha)}\mu}\right) = \left(\begin{array}{c} (1-a)\left(\frac{\mu}{1+\beta}\right)^{\frac{1}{1-\alpha}} p_t^{\frac{\alpha}{1-\alpha}} \\ +a\left(\frac{\mu}{1+\beta}\right)^{\frac{1}{(1-\alpha)\alpha}} p_t^{\frac{\alpha}{1-\alpha}}\left(\frac{\mu}{1+\beta}\right)^{-\frac{1-\alpha}{\alpha}}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \\ -(1-a)\, p_t^{-\alpha}\left(\frac{\alpha}{1-\alpha}\right)^{-\alpha}\chi^\alpha\phi \end{array}\right)$$

$\left(\frac{\mu}{1+\beta}\right)^{\frac{1}{(1-\alpha)\alpha}} p_t^{\frac{1}{1-\alpha}}\left[(1-a)\,\mu\left(\frac{\mu}{1+\beta}\right)^{-1} + a\alpha\right] - (1-a)(1-\alpha)\left(\frac{\mu}{(1+\beta)}\right)^{\frac{1-\alpha}{\alpha}} p_t^{1-\alpha}\chi^\alpha\phi -$
$(\beta(1-a)+\alpha) = 0$

The derivation above can be rearranged as

$\left[(1-a)(1+\beta)+a\alpha\right]\left(\frac{\mu}{1+\beta}\right)^{\frac{1}{(1-\alpha)\alpha}} p_t^{\frac{1}{1-\alpha}} - (1-a)(1-\alpha)\left(\frac{\mu}{(1+\beta)}\right)^{\frac{1-\alpha}{\alpha}} p_t^{1-\alpha}\chi^\alpha\phi - (\beta(1-a)+\alpha) =$
0

For a consistency check, we can put $p_t = \left(\frac{\mu}{1+\beta}\right)^{-\frac{1}{\alpha}}$ and $\chi = 1$ and $\phi = 1$

the polynomial function satisfies

$\left[(1-a)(1+\beta)+a\alpha\right]\left(\frac{\mu}{1+\beta}\right)^{\frac{1}{(1-\alpha)\alpha}}\left(\frac{\mu}{1+\beta}\right)^{-\frac{1}{\alpha(1-\alpha)}} - (1-a)(1-\alpha)\left(\frac{\mu}{(1+\beta)}\right)^{\frac{1-\alpha}{\alpha}}\left(\frac{\mu}{1+\beta}\right)^{-\frac{1-\alpha}{\alpha}} -$
$(\beta(1-a)+\alpha) = 0$

where $\mu = \left((1-\alpha)^{1-\alpha}\alpha^\alpha\right)$

6.4 D Simulation exercise for flex-price

Good market is in equilibrium where price p_t clears out the market in symmetric equilibrium (note that symmetry applies for m goods)

$y_t^d = y_t^s$

$$y_t^d = p_t^{\frac{\alpha}{1-\alpha}} p_{t-1}^{-\alpha} \left(\frac{aH}{mP_t} + a \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \left(\left(\frac{\alpha}{1+\beta}\right)^{\frac{\alpha}{1-\alpha}} - \left(\frac{\alpha}{1+\beta}\right)^{\frac{1}{1-\alpha}} \right) y_{t-2} + (1-\alpha) \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \chi^\alpha \phi p_t^{-\alpha} y_{t-1} \right)$$

$$y_t^s = p_t^{\frac{\alpha}{1-\alpha}} \left(\frac{\alpha}{1+\beta}\right)^{\frac{\alpha}{1-\alpha}} p_{t-1}^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} y_{t-2}$$

with parameters and market clearing conditions of the previous period

with parameters

$$\begin{array}{lll} \beta = 0.25 & a = 0.75 & y_{t-1} = 0.809\,728\,0 \\ m = 50 & H = 100 & p_{t-1} = 2.\,849\,960 \\ \alpha = 0.75 & \mu = \left((1-\alpha)^{1-\alpha} \alpha^\alpha\right) & \chi = 1, \;\; \phi = 1 \end{array}$$

Shocks at time t ($\phi_t = 1.05$)

From previous periods

$$p_{t-1} = \left(\frac{\mu}{1+\beta}\right)^{-\frac{1}{\alpha}} \quad \mu = \left((1-\alpha)^{1-\alpha} \alpha^\alpha\right) \quad y_{t-2} = y_{t-1} = \left(\frac{\mu}{(1+\beta)}\right)^{\frac{1-\alpha}{\alpha}} \frac{aH\mu}{m(\beta(1-a)+\alpha)}$$

6.5 E The Derivation of the Pricing Rule for Fix-Price Equilibrium

Pricing rule is similar to flex price equilibrium with the exception that in the formation of the demand, some part of the demand schedule of workers does not become effective as they had planned. To make the notation more easy, we continue from the section of step-by-step iteration of fix price equilibrium. Time subscripts can be dropped since the equilibrium starts from the point where there is no conjectural change of expectations. Returning again equation (22), with the assumption that $\beta = 0$ which refers to the competitive market case

$$p_t = \frac{w_t \left(\frac{aHw_{t-1}}{mp_{t-1}} + \left(\frac{p_{t-1}}{\chi^e w_t}\right)^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \phi^e y_{t-1} \right)^{\frac{1}{\alpha}} \left(\left(\frac{p_{t-2}}{w_{t-1}}\right)^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} y_{t-2} \right)^{\frac{-(1-\alpha)}{\alpha}} + p_{t-1} \left(\frac{p_{t-1}}{\chi^e w_t} \frac{\alpha}{1-\alpha}\right)^{-\alpha} \phi^e y_{t-1}}{\frac{aHw_{t-1}}{mp_{t-1}} + \left(\frac{p_{t-1}}{\chi^e w_t}\right)^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \phi^e y_{t-1}}$$

Dropping the time subscripts and setting $\phi^e \neq 1$ and $\chi^e \neq 1$

$$p = \frac{Y^d}{Y^c} = \frac{wy^{\frac{1}{\alpha}} \left(\left(\frac{p}{w}\right)^{-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} y \right)^{\frac{-(1-\alpha)}{\alpha}} + p \left(\frac{p}{w} \frac{\alpha}{1-\alpha}\right)^{-\alpha} y}{y} = \frac{w^\alpha y p^{1-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} + w^\alpha y p^{1-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} y}{y}$$

$$p = w^\alpha p^{1-\alpha} \left(\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} + \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \right)$$

$$\frac{p}{w} = \mu^{-\frac{1}{\alpha}}$$

where $\mu = \left((1-\alpha)^{1-\alpha}\,\alpha^{\alpha}\right)$

We can easily replace the pricing rule and get the equations, which are imposed to derive the equation (23).

$y_{t-2} = y_{t-1} = \mu^{\frac{1}{\alpha}} \frac{\grave{a}H}{m\alpha} \quad \mu = \left((1-\alpha)^{1-\alpha}\,\alpha^{\alpha}\right) \frac{p_{t-1}}{w_{t-1}} = \left(\frac{1}{\mu}\right)^{\frac{1}{\alpha}}$ where $w_t = w_{t-1} = 1$

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